

## Magnetization of bilayer two-dimensional electron systems

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**Abstract.** We have measured the magnetization of strongly coupled bilayer two-dimensional electron systems. When sweeping the magnetic field steps appear in the magnetization whenever a transition between discrete energy levels takes place. At magnetic fields for which the Landau-level splitting dominates, the steps occurring at total filling factors  $\nu_T = 4j$  are related to transitions between adjacent Landau levels  $j - 1$  and  $j$ ; such steps can also be observed in single layers. Additional magnetization steps showing up in bilayers at half Landau-level filling  $\nu_T = 4j + 2$  are associated with transitions from a symmetric to an antisymmetric state inside the same Landau level  $j$ . The observed size of the Landau-level steps in bilayers is considerably lower than expected theoretically. These findings are explained using a model with a large background density of states.

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**1. Introduction**

The magnetization of an electron system is one of the few physical properties which can directly access its ground state. In particular, magnetization experiments give an insight into the single-particle energies as well as correlation effects. In semiconductor-based two-dimensional electron systems (2DESs), the first efforts of measuring the magnetization were focussed on single layers [1]–[5]. Only recently the magnetization of more complex systems also exhibiting electron-correlation effects were investigated [6]–[8]. In this paper, we present experiments on the magnetization of strongly coupled bilayer 2DESs. We will show that we can visualize transitions between Landau levels as well as intra-Landau-level electronic transitions from a symmetric to an anti-symmetric state as steps in the magnetic-field dependence of the magnetization. Using a model involving a finite background density-of-states (DOS) [9] we will explain the low magnetization steps for Landau-level transitions.

**2. Magnetization***2.1. Magnetization and torque*

In most general terms the magnetization  $\vec{M}$  of an electron system with constant particle number  $N$  is defined from its free energy  $F$  as

$$\vec{M} = - \left. \frac{\partial F}{\partial \vec{B}} \right|_N. \quad (1)$$

This magnetization leads to a torque on the sample given by

$$\vec{T} = \vec{M} \times \vec{B} + \vec{d} \times \vec{\nabla}(\vec{M} \cdot \vec{B}). \quad (2)$$

The first term,  $\vec{M} \times \vec{B}$ , represents the torque of an anisotropic magnetic moment with a component perpendicular to the applied magnetic field. The second term arises from a magnetic force  $\nabla(\vec{M} \cdot \vec{B})$  on the sample; it is only present when the centre of mass is situated at a distance  $d$  from the torque axis and when the sample is placed in a field gradient.

## 2.2. Magnetization of a two-dimensional (2D) electron system

A quantizing magnetic field  $B_n$  perpendicular to the 2DES leads to the formation of Landau levels at discrete energies,  $E_j = (j + \frac{1}{2})\hbar\omega_c$ , where  $\omega_c = eB_n/m^*$  is the cyclotron frequency for electrons with an effective mass  $m^*$ . In a bilayer, each Landau level is split into a symmetric and an antisymmetric state with a symmetric–antisymmetric energy splitting  $\Delta_{\text{SAS}}$ . In GaAs the additional spin splitting of these levels is so small that it will be neglected for simplicity in the following. When increasing the magnetic field, the levels are successively depopulated. At transition points where one level becomes totally empty the field dependence of the free energy changes leading to a step in the magnetization perpendicular to the 2DES. The step size  $\Delta M_{\perp}$  is given by the Maxwell relation

$$\Delta M_{\perp} = \frac{N}{B_n} \Delta\mu. \quad (3)$$

Here  $\Delta\mu$  is the change of the chemical potential at the transition point and  $N$  is the total number of electrons.

For a bilayer 2DES with an additional degree of freedom into the third dimension the free energy also depends on the in-plane magnetic field  $B_{\parallel}$ . With equation (1) this leads to an additional in-plane magnetization  $M_{\parallel}$ . However, since the Landau-level occupancy only depends on the perpendicular field component, the in-plane magnetization does not display any discontinuity.

In our experiments on 2DESs with a strongly anisotropic magnetization the sample is placed symmetrically onto the torque wire in the centre of the field. Due to the magnetization anisotropy a finite contribution  $\vec{M} \times \vec{B}$  can be measured. The torque resulting from it can be written in two components as:

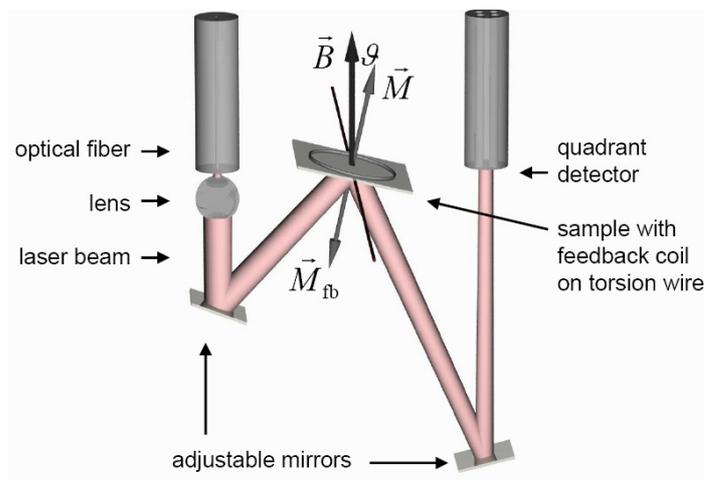
$$T = M_{\perp} B \sin \vartheta + M_{\parallel} B \cos \vartheta, \quad (4)$$

where  $\vartheta$  is the angle between the 2DES normal and the applied magnetic field. The first term describes the magnetization perpendicular to the 2DES; it contains both magnetization steps arising from discontinuities in the chemical potential as well as a continuous background. The second term, originating from magnetization components in the 2DES-plane, is fully continuous.

## 3. Experimental set-up

### 3.1. Torque magnetometer with optical detection

The precise working principle of our magnetometer is illustrated in figure 1 [10]. A 2DES sample is mounted onto a torque wire with a finite angle  $\vartheta$  between the sample normal and the direction of the magnetic field. As described in equation (4) a magnetization of the sample leads to a torque which results into a small rotation of the sample around the torque axis. The sample rotation is detected optically by focusing a laser beam reflected on the backside of the sample onto



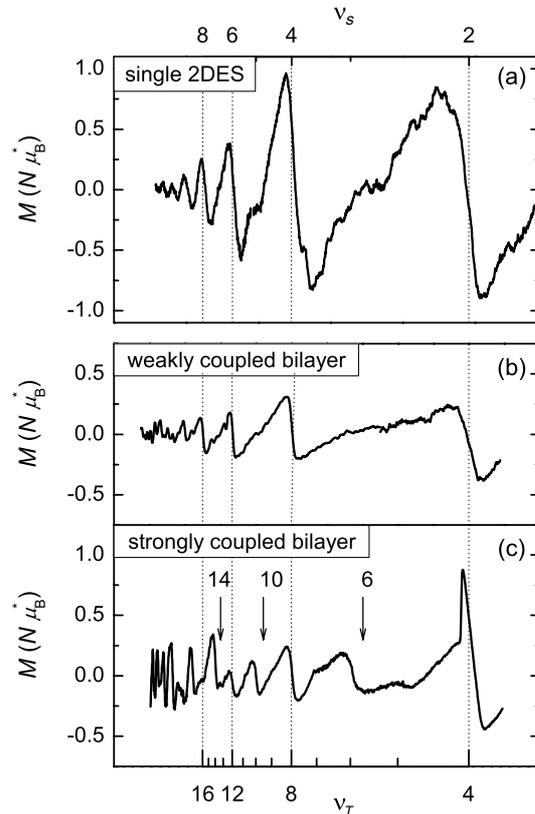
**Figure 1.** Schematic set-up of the torque magnetometer with optical detection.

a quadrant detector. In order to keep the sample at a fixed angle we counteract the torque using the magnetic moment  $M_{fb} = IA$  of a feedback coil glued on the back of the sample ( $I$  is the current through the feedback coil and  $A$  its cross-sectional area). Discontinuities in  $M_{\perp}$  will directly show up as steps in  $M_{fb}$ ,  $\Delta M_{\perp} = -\Delta M_{fb}$ . Additionally,  $M_{fb}$  contains a continuous contribution arising from the continuous part in  $M_{\perp}$ , from  $M_{\parallel}$  and from the substrate magnetization. Operation in feedback mode also allows to damp mechanical noise in the set-up by means of an active PID-control which considerably enhances the experimental sensitivity.

### 3.2. Samples

In this paper, we present the magnetization of two bilayer 2DESs, one with a weak and one with a strong interlayer coupling. The samples consist of two 10 nm wide GaAs quantum wells embedded in  $\text{Ga}_{0.65}\text{Al}_{0.35}\text{As}$ . Electrons are provided by modulation doping the left and right barriers symmetrically with Si. Magnetotransport experiments on reference bilayer samples with similar layer sequence and doping and equipped with a top gate, have shown that the layers of our ungated sample are well balanced. In the weakly coupled bilayer a 4 nm barrier separates the two quantum wells, the total electron concentration is  $n = 9.2 \times 10^{11} \text{ cm}^{-2}$  and the Hall mobility  $4.2 \times 10^5 \text{ cm}^2 \text{ Vs}^{-1}$ . The sample area is  $0.37 \text{ cm}^2$ . The strongly coupled bilayer has a 2.5 nm barrier, a total electron concentration of  $n = 7.4 \times 10^{11} \text{ cm}^{-2}$ , Hall mobility of  $1.9 \times 10^5 \text{ cm}^2 \text{ Vs}^{-1}$  and sample area  $0.50 \text{ cm}^2$ . The concentrations and mobilities were determined at the actual experimental conditions in zero field, i.e. at low temperature after illumination with an infrared light-emitting diode until saturation was reached. Transport experiments were performed to confirm that the laser light from the optical read-out scheme has no effect on the 2D electron system. Self-consistent calculations yield estimates of the symmetric–antisymmetric splitting in the balanced bilayers:  $\Delta_{\text{SAS}} \approx 1.5 \text{ meV}$  for the weakly coupled system and  $\Delta_{\text{SAS}} \approx 3 \text{ meV}$  for the strongly coupled bilayer.

As a reference we compare the bilayer magnetization to that of a high-mobility single-layer 2DES in a GaAs-AlGaAs heterojunction with an electron concentration  $n = 4.8 \times 10^{11} \text{ cm}^{-2}$ , mobility  $2.2 \times 10^6 \text{ cm}^2 \text{ Vs}^{-1}$  and sample area  $0.56 \text{ cm}^2$  [11].



**Figure 2.** Magnetization of a weakly coupled bilayer 2DES (b) and a strongly coupled bilayer (c) compared to a single-layer 2DES (a) at a temperature  $T = 1.2$  K. Sample details are given in subsection 3.2.

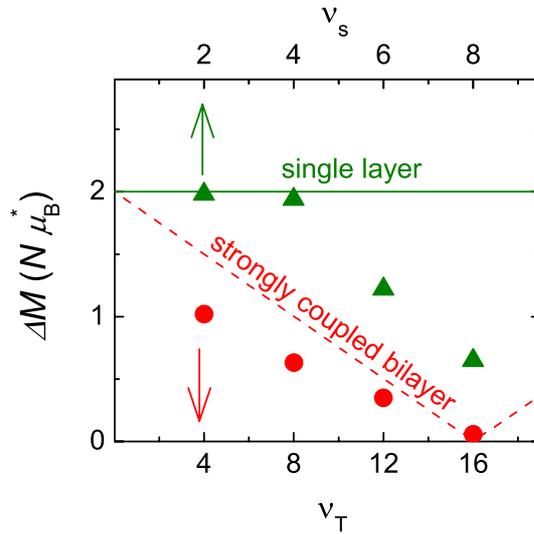
## 4. Experimental results and discussion

### 4.1. Results

Figures 2(b) and (c) show the magnetization of the two bilayer samples (after subtraction of a smooth background) as a function of the filling factor  $\nu = nh/eB_{\perp}$  at  $T = 1.2$  K compared to a single-layer 2DES plotted in figure 2(a). The samples were mounted at an angle  $\vartheta = 13^{\circ}$ . The magnetization, expressed as the magnetic moment per electron, is plotted in units of the effective Bohr magneton  $\mu_B^* = (\hbar e)/(2m^*)$ .

Transitions of the chemical potential between two Landau levels show up as magnetization steps at filling factors  $\nu_T = 4j$  for the bilayers and  $\nu_s = 2j$  for the single layer. For the bilayer additional magnetization steps related to transitions between the symmetric and antisymmetric state within the same Landau level appear at  $\nu_T = 4j + 2$ . In the weakly coupled bilayer  $\Delta_{SAS}$  is just strong enough to visualize these transitions as shoulders in the magnetization signal; in the strongly coupled system clear steps are visible. These observations clearly show that it is indeed possible to observe discontinuities in the thermodynamic properties of an electronic state as measurable magnetization steps.

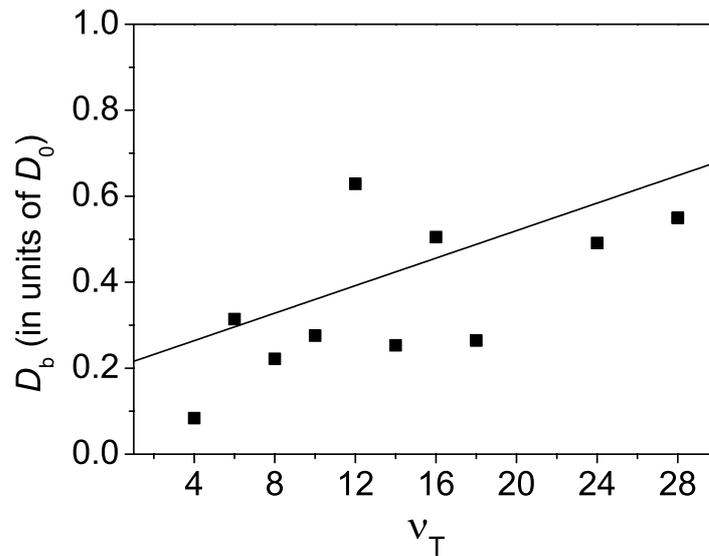
For the single layer the step size of the Landau-level steps saturates at the theoretically expected  $\Delta M = 2N\mu_B^*$  for low filling factors (see also [12]). For higher filling factors the



**Figure 3.** Magnetization steps at Landau-level transitions  $\nu_T = 4, 8, 12$  and  $16$  for the strongly coupled bilayer (●, bottom axis) compared to the Landau-level steps in a single layer at  $\nu_s = 2, 4, 6$  and  $8$  (▲, top axis). The lines indicate the step sizes as theoretically expected for these filling factors.

experimentally observed step sizes are reduced due to finite-temperature effects and level broadening. In contrast, the corresponding step sizes for the bilayer 2DESs saturate at a considerably smaller value,  $\Delta M < N\mu_B^*$  (an additional peak-feature appearing around  $\nu = 4$  will be discussed further on). Part of this reduction can be explained by the fact that the transition takes place from the symmetric state of a higher Landau level to the antisymmetric state of the Landau level below. The jump in the chemical potential is not  $\hbar\omega_c$ , but rather  $\hbar\omega_c - \Delta_{\text{SAS}}$  which is expected to lead to a correspondingly reduced magnetization step  $\Delta M = 2N\mu_B^*(\hbar\omega_c - \Delta_{\text{SAS}})/\hbar\omega_c$ . In particular, for the strongly coupled bilayer we observe a disappearance of the Landau-level step at  $\nu = 16$ . When supposing  $\hbar\omega_c = \Delta_{\text{SAS}}$  at this particular filling factor this yields indeed a quite realistic value of  $3.3$  meV for  $\Delta_{\text{SAS}}$ . At higher filling factors  $\Delta_{\text{SAS}}$  exceeds  $\hbar\omega_c$ , which complicates the energy-level scheme: steps across the symmetric-antisymmetric gap ( $\nu = 20, 24$  and  $28$ ) are now associated with an energy gap of  $\Delta_{\text{SAS}} - \hbar\omega_c$ , and the Landau-level steps ( $\nu = 18, 22$  and  $26$ ) with  $\Delta\mu = 2\hbar\omega_c - \Delta_{\text{SAS}}$ .

The magnetization steps between two Landau levels at lower filling factors  $\nu = 12, 8$ , and  $4$  are much stronger reduced than expected from  $\Delta M = 2N\mu_B^*(\hbar\omega_c - \Delta_{\text{SAS}})/\hbar\omega_c$  (broken line in figure 3). This observation is summarized in figure 3 where we compare the experimentally measured magnetization steps at Landau-level transitions for the strongly coupled bilayer and the single layer to the theoretically expected behaviour as described above. The experimental step size at filling factor  $\nu$  is extracted from the data by extrapolating the sloping magnetization traces before and after the step and measuring their difference  $\Delta M$  at the position of  $\nu$ . For the single layer the experimental values for low enough filling factors, i.e.  $\nu = 2, 4$  indeed fall on the (solid) line; for the bilayer the step sizes are clearly smaller than naively expected: the reduction becomes even more pronounced for lower filling factors. We have verified that the step sizes for the lowest two Landau level steps no longer depend on temperature for  $T < 2$  K; thermal broadening can therefore be excluded to be responsible for the observed reduction.



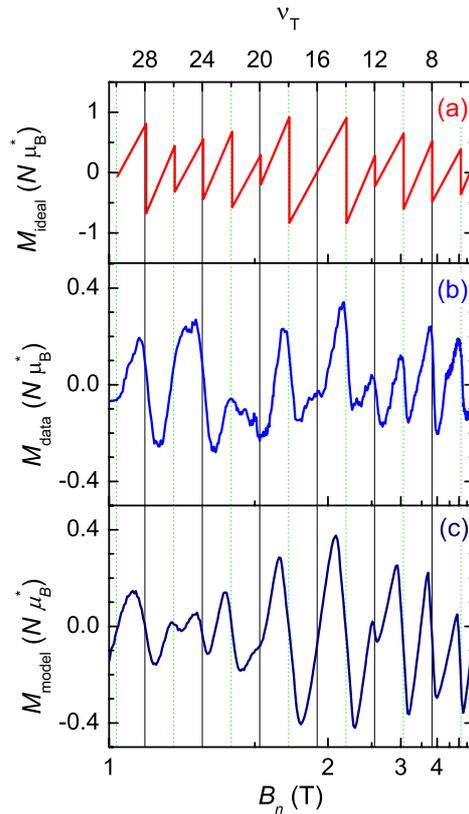
**Figure 4.** Average DOS at the Fermi level at even filling factor derived from the magnetization data for the strongly coupled bilayer. The line indicates the background DOS model applied in our calculations:  $D_b = (0.20 + 0.016\nu_T)D_0$ .

#### 4.2. Model calculations

In order to explain this considerable reduction of experimentally observed magnetization steps at Landau-level transitions, we use a phenomenological DOS model with Gaussian-broadened Landau levels and a finite background DOS [9] assuming balanced layers. As model parameters for sample 2 we use  $\Delta_{SAS} = 3.3$  meV (see above), a constant Landau-level broadening  $\Gamma = 0.2$  meV and a filling-factor dependent background DOS  $D_b = (0.20 + 0.016\nu_T)D_0$  ( $D_0$  is the DOS at zero magnetic field for one symmetry state including the two-fold degeneracy of spin). In figure 4 this background DOS model is plotted together with the DOS values derived from the width  $\Delta\nu$  and size  $\Delta M$  of the observed magnetization steps following the procedure described in [9]. For simplicity, we chose to model the background DOS by a straight line; to improve the agreement between the magnetization data and the calculations we had to position this line at slightly larger background DOS values than would be expected from the data points (■) in figure 4. In figure 5, we compare our magnetization calculations to the experimental results and to an ideal bilayer with no level broadening and no background DOS. The expectation for an ideal bilayer predicts clearly far too large Landau-level steps in the magnetization. Our simple model calculation, however, reasonably reproduces the experimentally observed results, although not in detail.

#### 4.3. Discussion

We have shown that it is necessary to include a large background DOS in order to explain the strongly reduced magnetization steps at Landau-level transitions in bilayers. As pointed out by e.g. [9], the reduction of magnetization steps by the presence of a background DOS can be related to the sample quality: a higher mobility sample is expected to have a lower background DOS. Since in coupled bilayer 2DESs electron–electron correlations might induce new effects, like



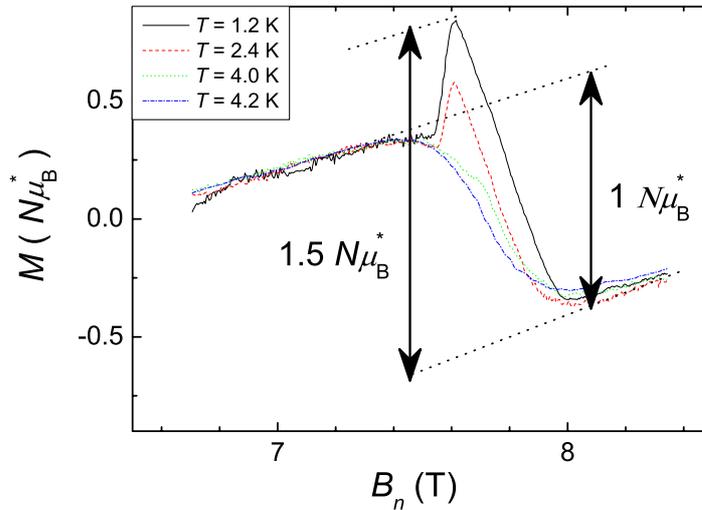
**Figure 5.** Comparison of the magnetization of an ideal bilayer (a) with the experimentally measured magnetization of the strongly coupled bilayer (b) and our model calculations (c). The data are plotted as a function of the field component perpendicular to the 2DES,  $B_n$  (bottom axis, reciprocal scale) and the total filling factor (top axis). Note the different y-axis scale for (a) compared to (b) and (c).

those observed recently in the two-valley system of Si/SiGe heterostructures [8], it is tempting to propose that inter-layer or inter-valley coupling naturally introduces a considerable background DOS in the equivalent single-particle spectrum: due to the large coupling between different energy levels a number of electrons can no more be assigned to a specific level and participate in a large background DOS.

Further experiments with samples of higher mobility and while varying the inter-layer coupling by an adjustable in-plane magnetic field may help to address this interesting question more definitely.

#### 4.4. Correlation effects at $\nu = 4$ ?

As briefly mentioned in section 4.1 an additional peak in the magnetization appears around  $\nu_T = 4$  when the  $j = 1$  Landau level is depopulated. Here the chemical potential jumps from a symmetric spin-up state of the first Landau level to an antisymmetric spin-down state in the lowest Landau level. The appearance of this peak is shown in more detail in figure 6. For high temperatures  $T > 4$  K solely a magnetization step of  $1 N\mu_B^*$  is visible. As discussed above, the



**Figure 6.** Magnetization of the strongly coupled bilayer around filling factor  $\nu = 4$  for different temperatures.

step size is reduced considerably by DOS effects. When cooling down to lower temperatures an additional peak in the magnetization signal appears just above  $\nu_T = 4$ . The peak does not depend on the direction of the magnetic-field sweep and, therefore, it cannot be due to eddy currents [9, 13]. Furthermore, because the bilayer is well balanced, also recharging effects [14] are improbable to explain the occurrence of the structure. The drastic dependence on temperature, however, hints to a correlation-induced cause for the appearance of this peak.

The final total step size reaches a value around  $1.5 N\mu_B^*$  which is indeed very close to the theoretically expected value for an ideal bilayer with no background DOS and broadening. This observation strongly proposes that the background DOS disappears just before the  $\nu_T = 4$  transition. In other words, the coupling of the few electrons remaining in the first Landau levels weakens and the background DOS is reduced.

More correlation effects may play a role and require further experiments with adjustable inter-layer coupling.

## 5. Conclusion

In conclusion, we have directly visualized the complex energy-level structure of strongly coupled bilayer 2DESs by measuring their magnetization. Magnetization steps were related to intra-Landau-level transitions of the chemical potential between bilayer states of different symmetry and to transitions between two different Landau-levels, intuitively showing the fundamental thermodynamic nature of magnetization.

In contrast to theoretical expectations for a bilayer, the magnetization steps at inter-Landau-level transitions are reduced considerably which we were able to explain by involving a large background DOS. At the transition to the lowest Landau level this background DOS seemed to disappear resulting into an additional peak observed in the magnetization.

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